Analysis of extremes in temperature across Iceland

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Outline

- Data on extremes at different locations :
  annual minimum temperature in Iceland
  annual maximum temperature in Iceland
- A model for extreme values: the generalized extreme value distribution
- A Bayesian hierarchical model for point-referenced extremes
- Results

Introduction

- Modeling the extremes in temperature is of interest to scientists, engineers and policy-makers.
- The method introduced here provides:
  - spatial mapping of T-year events
  - estimation of time trend

Data

- Variable 1 of interest: the minimum temperature from July 1st to June 30th.
- Variable 2 of interest: the maximum temperature from January 1st to December 31st.
- 72 sites in Iceland, $N = 72$.
- 45 years, $T = 45$.
- Data from 1961 to 2006.
- About 32 observations on average per site.

The gen. extreme value distribution

- The appropriate distribution to describe block maxima and minima. Based on a theoretical background.
- In case of minima the cdf is

$$F(y) = 1 - \exp\left[-\left\{1 - \xi\left(\frac{y-\mu}{\sigma}\right)\right\}^{-1/\xi}\right]$$

if $\xi > 0,$ then $y < \mu + \sigma/\xi$ (b. f. above)

if $\xi < 0,$ then $y > \mu + \sigma/\xi$ (b. f. below)

The gen. extreme value distribution

- $-\infty < \mu < \infty, \quad \sigma > 0, \quad -\infty < \xi < \infty$
- A special case, $\xi = 0$

$$F(y) = 1 - \exp\left[- \exp\left\{ \left(\frac{y - \mu}{\sigma} \right) \right\} \right]$$

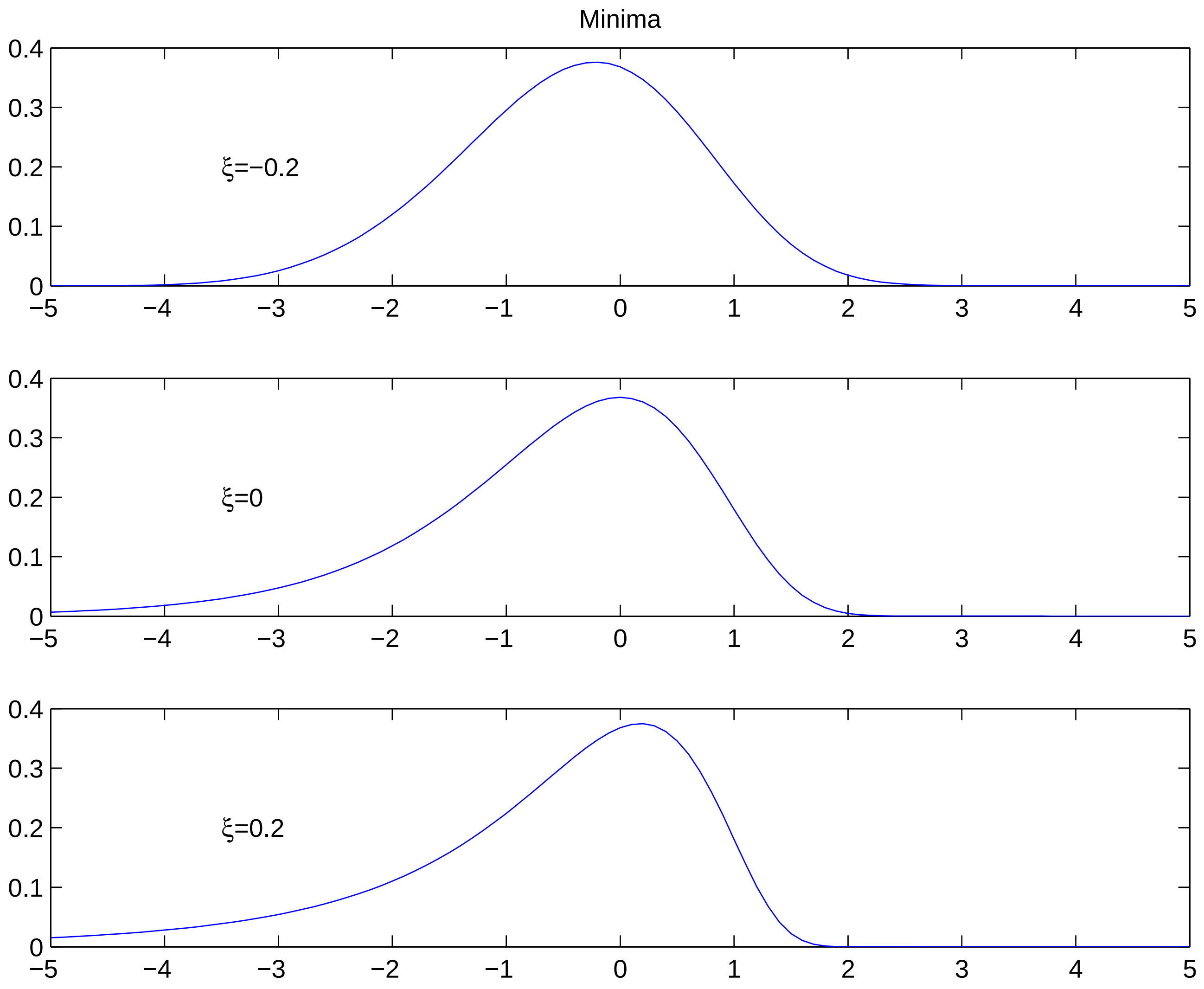
$$-\infty < y < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$$

- Referred to as the extreme value distribution of type I or as the Gumbel distribution for minima.

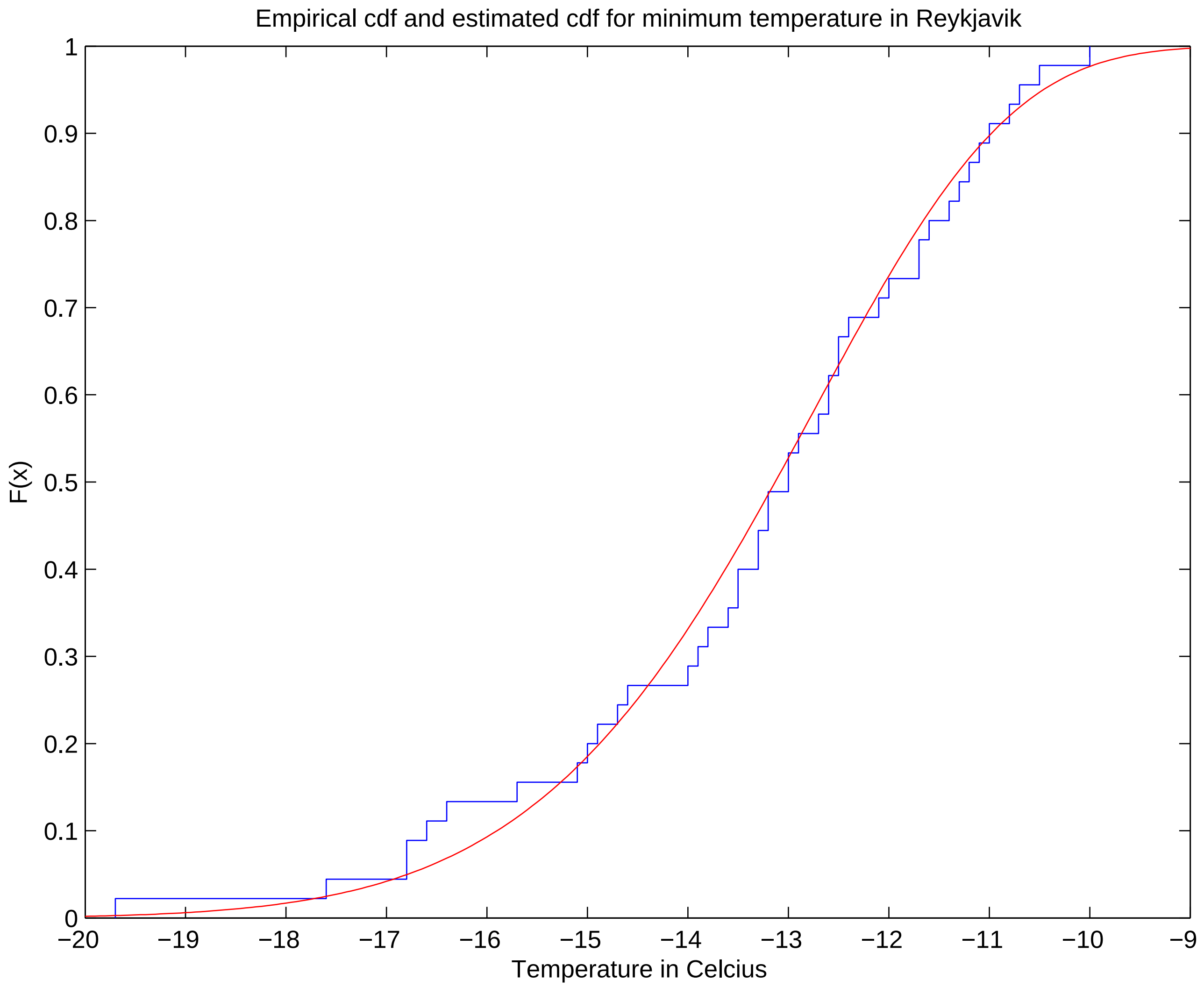
The gen. extreme value distribution

- μ : location parameter
- σ : scale parameter
- ξ : shape parameter
- ξ : hard to estimate

The density of the gen. ext. val. dist.



The empirical cdf and estim. cdf, Reykjavik



A model for extreme temperature

- $y_{jt,\mathrm{obs}}$: Extreme temperature observed at site j and time t, $j = 1, ..., N$, $t = 1, ..., T$
- μ : vector of location parameters, dim. N
- τ : vector of log-scale parameters, dim. N
- ξ : the shape parameter (same for all sites)
- Use spatial models for μ and τ.

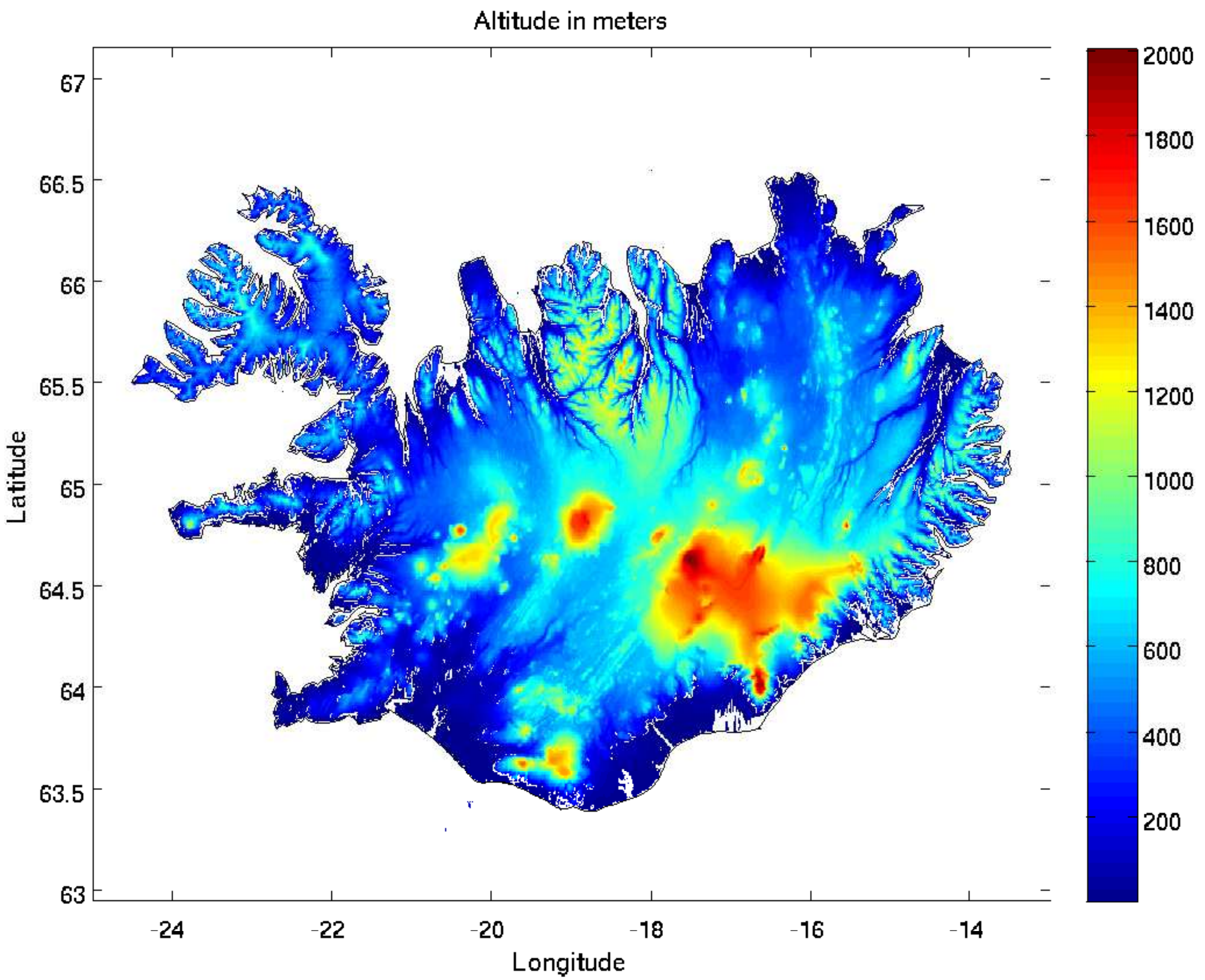
The design matrices

- Fixed effects
  - $X_\mu : \; X_{\mu,1,j} = 1$
  - $X_{\mu,2,j} = y_j$ (latitude at site j)
  - $X_{\mu,3,j} = z_j$ (altitude at site j)
  - $X_{\mu,4,j} = v_j$ (the shortest distance from site j to open sea)
  - $X_\tau : \; X_{\tau,1,j} = 1$

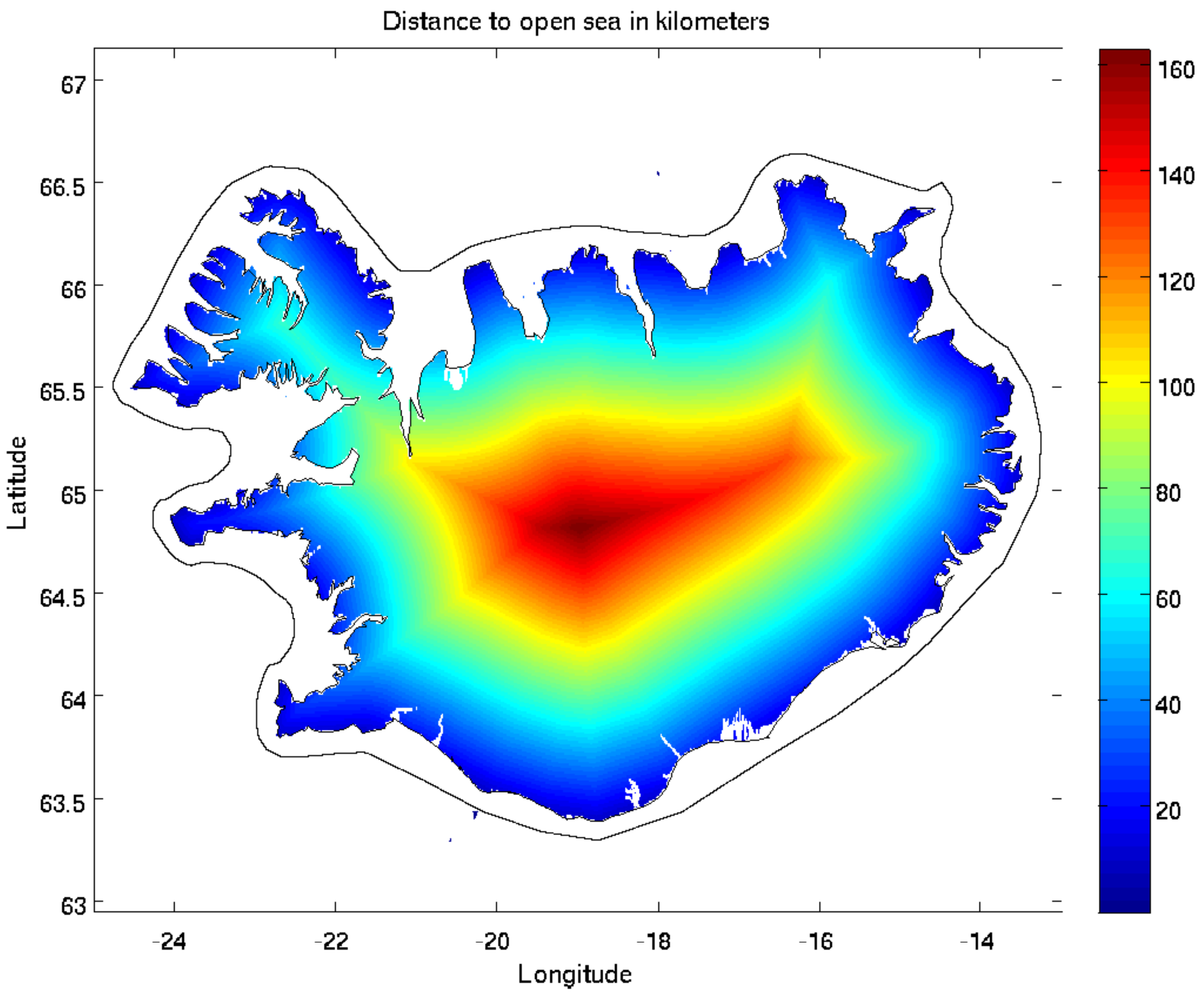
The design matrices

- Random effects
  - Z_μ is a matrix based on 60×42 two dimensional cubic B-spline functions.
  - Z_τ is a matrix based on 30×21 two dimensional cubic B-spline functions.

Altitude


Altitude in meters
67
66.5
66
65.5
65
64.5
64
63.5
63
Latitude
-24
-22
-20
-18
-16
-14
Longitude
2000
1800
1600
1400
1200
1000
800
600
400
200

Distance to open sea



A model for extreme temperature

- $y_{t,\text{obs}} \sim$ $\text{Gtgev}(\mu + \Delta_0(t - t_0), \exp(\tau), \xi, R_y(\phi_y, \nu_y))$ $t = 1, ..., T$
- $R_y(\phi_y, \nu_y)$: Matérn correlation matrix
- $\mu \sim \mathsf{N}(X_\mu \beta + Z_\mu a, \sigma_\mu^2 I)$
- $\tau \sim \mathsf{N}(X_\tau \eta + Z_\tau b, \sigma_\tau^2 I)$
- $a \sim \mathsf{N}(0, \kappa_\mu^2 (I_\mu - \phi_\mu C_\mu)^{-1} M_\mu),\ \sum_k \sum_l a_{kl} = 0$
- $b \sim \mathsf{N}(0, \kappa_\tau^2 (I_\tau - \phi_\tau C_\tau)^{-1} M_\tau),\ \sum_k \sum_l b_{kl} = 0$

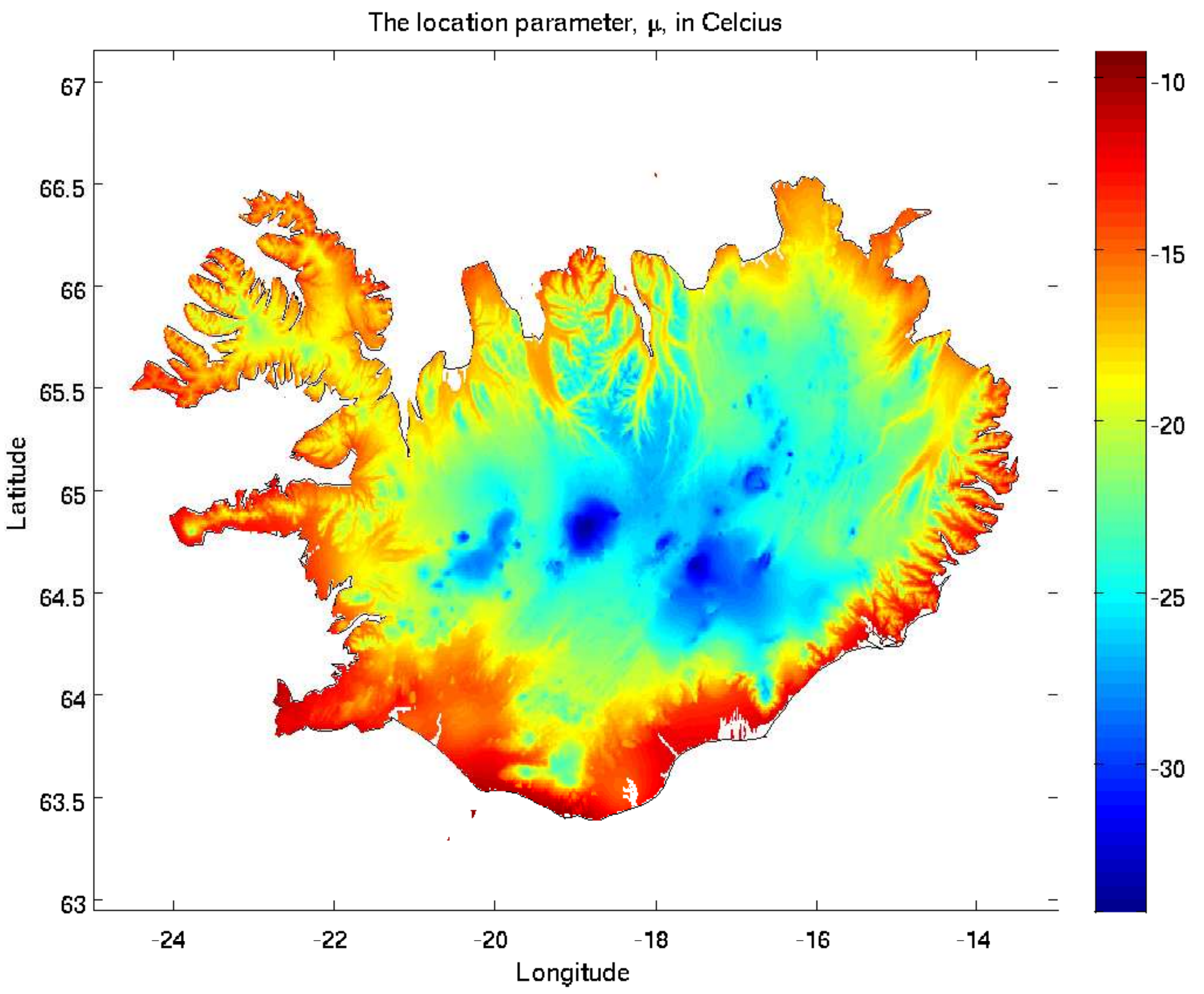
Estimates of parameters

- Minimum temperature
  - shape : $\xi = -0.14 \quad (-0.17, -0.11)$
  - trend : $10\Delta_0 = 0.48 \quad (0.24, 0.70)$
  - latitude : $\beta_2 = -0.98 \quad (-2.05, 0.28)$
  - altitude : $\beta_3 = -0.73 \quad (-1.89, 0.30)$
  - dist.sea : $\beta_4 = -6.30 \quad (-10.33, -1.62)$

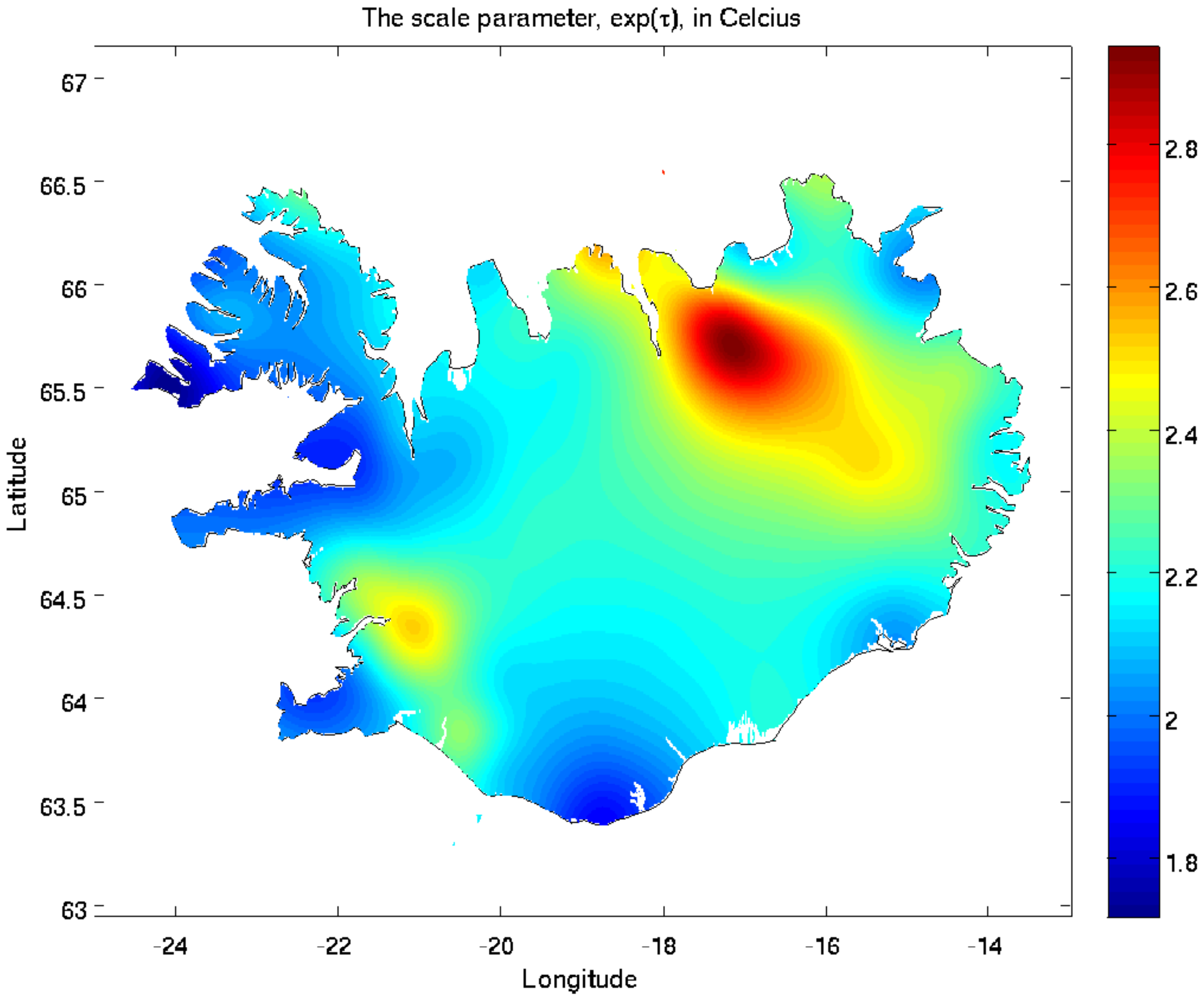
Estimates of parameters

- Maximum temperature
  - shape : $\xi = -0.12 \quad (-0.15, -0.08)$
  - trend : $10\Delta_0 = 0.38 \quad (0.18, 0.57)$
  - latitude : $\beta_2 = 0.21 \quad (-0.61, 1.21)$
  - altitude : $\beta_3 = -0.66 \quad (-1.51, 0.13)$
  - dist.sea : $\beta_4 = 4.16 \quad (0.94, 7.41)$

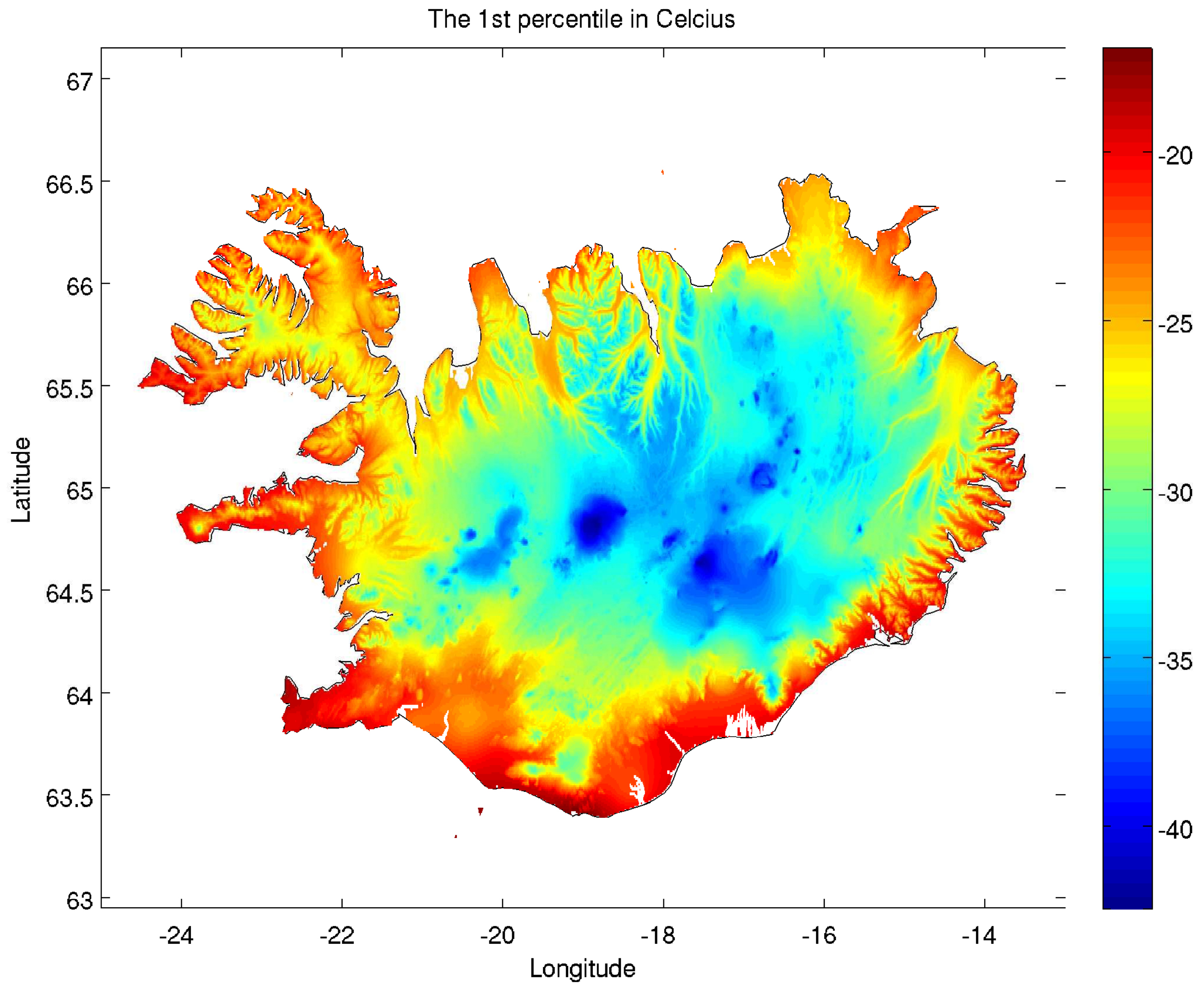
The post. median of μ



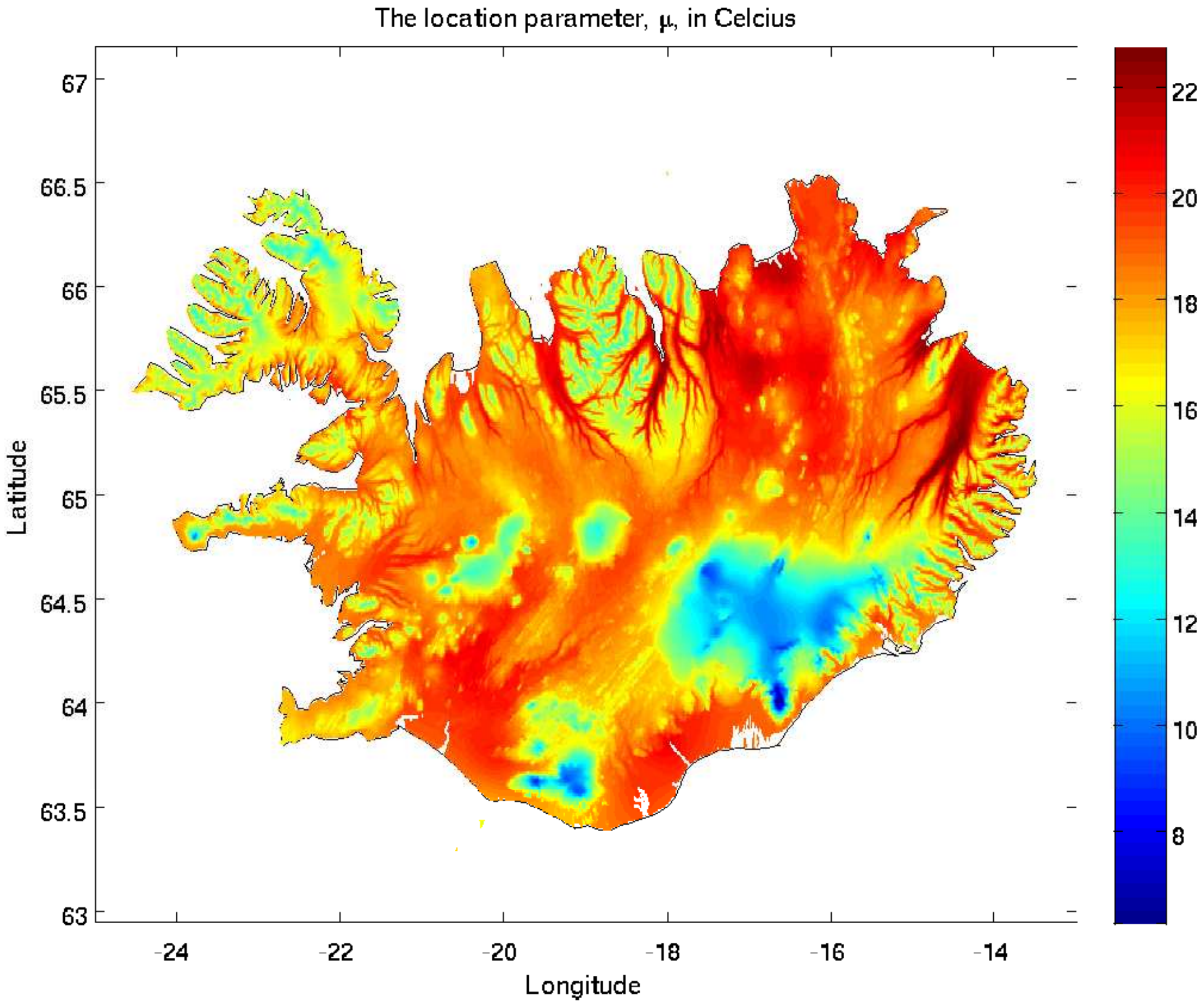
The post. median of $\sigma = \exp(\tau)$



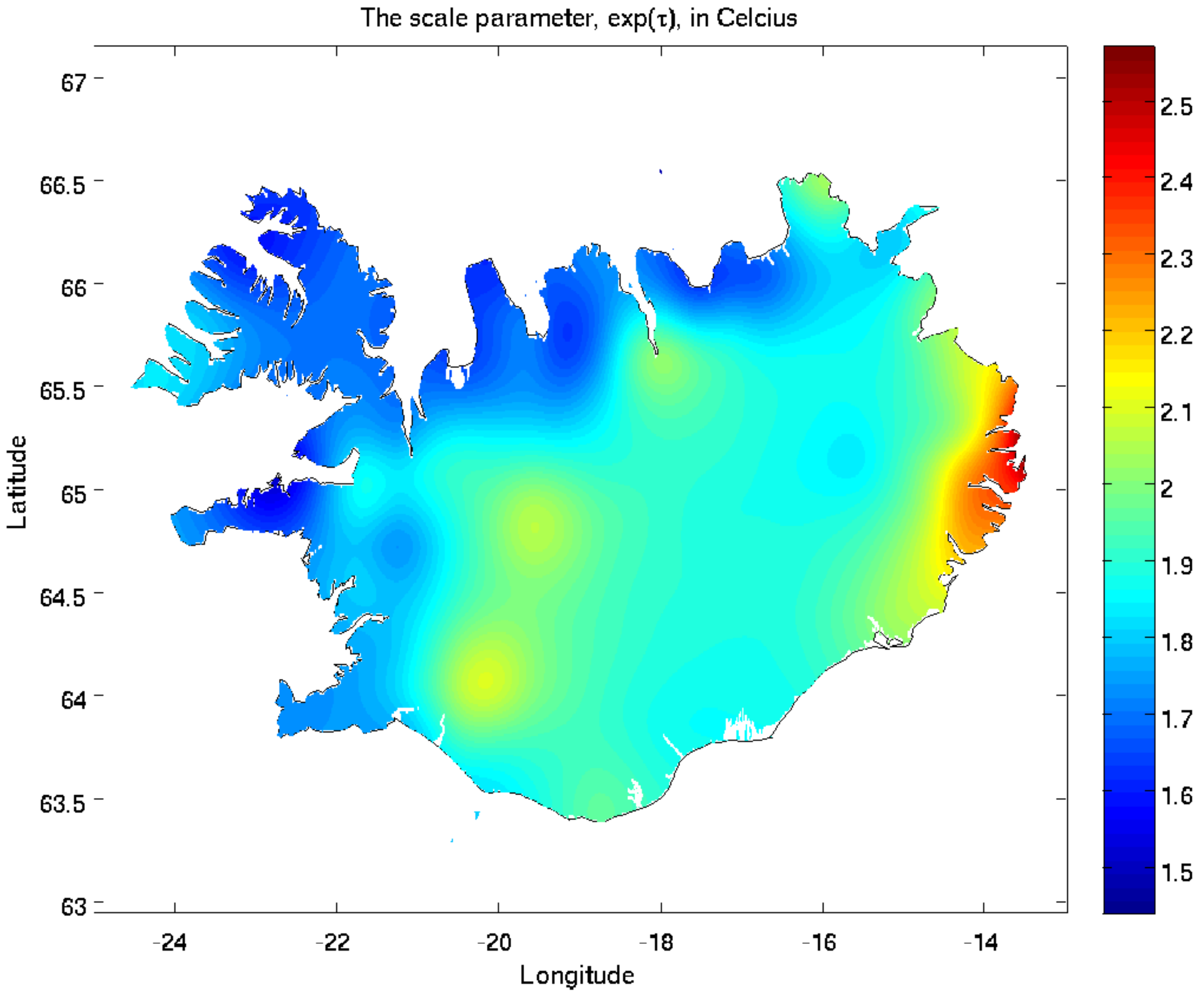
The post. median of the 1st percentile



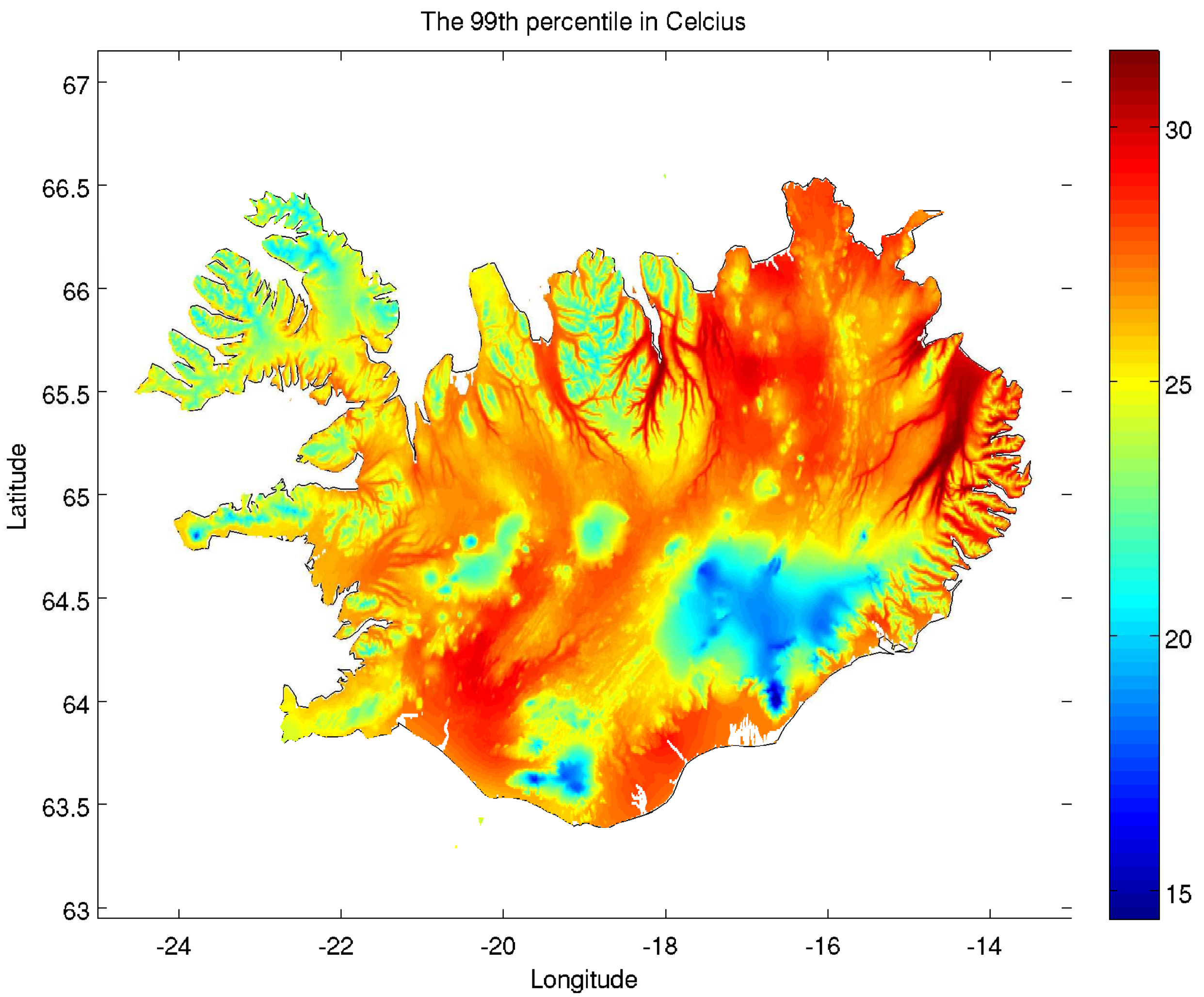
The post. median of μ



The post. median of $\sigma = \exp(\tau)$



The post. median of the 99th percentile



Time trend min & max