Modelling annual maximum 24 hour precipitation in Iceland using block sampling and SPDE models

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1 Introduction

2 Model structure

3 Results

4 Conclusions
The main goal is to obtain distributional properties of extreme precipitation on a high resolution grid over Iceland.

We rely on

- Observations of 24 hour annual extreme precipitation from 86 sites over the years 1961 to 2006
- Covariates based on outputs from a linear model of orographic precipitation on a 1 km$^2$ regular grid [Crochet et al., 2007]

A method for spatial quantile predictions of extreme precipitation combining these two sources is presented.
I HAVE IN MY POSSESSION A MAP (OF ICELAND)
OBSERVATION STATIONS AND TIME SERIES
The data are modelled with a Bayesian hierarchical model assuming a generalized extreme value distribution for the observations.

That is, let $y_{it}$ denote the annual maximum 24 hour precipitation at station $i$ at year $t$, with a cumulative distribution function of the form

$$F(y_{it}) = \exp \left\{ - \left( 1 + \xi \left( \frac{y_{it} - \eta_i}{\sigma_i} \right) \right)^{-1/\xi} \right\},$$

where $\eta_i$, $\sigma_i$ and $\xi$ are location, scale and shape parameters, respectively.
Working within the LGM framework, decompose $\eta$ into

$$\eta = X_\eta \beta_\eta + A u_\eta + v_\eta,$$

(1)

where

- $X_\eta$ is a design matrix based on outputs from the local climate model
- $\beta_\eta$ is the corresponding weight
- $u_\eta$ denotes a Matérn type spatial field constructed with the SPDE approach of [Lindgren et al., 2011]
- $A$ is a projection matrix
- $v_\eta$ is an unstructured random effect.

We assign the following priors

- $\beta_\eta \sim \mathcal{N}(0, \kappa^{-1}_\beta)$
- $u_\eta \sim \mathcal{N}(0, (Q_u(\tau_\eta, \theta_\eta))^{-1})$
- $v_\eta \sim \mathcal{N}(0, \kappa^{-1}_v I)$.

Similar structure is implemented for $\nu_i := \log \sigma_i$.
Reasonable covariates for extreme precipitation should contain information such as

- The topology of the domain
- Underlying physical processes
- Preferably on a high resolution grid

It has been suggested (Benestad et al, 2012)\cite{7} that observed mean values are useful predictors for extreme precipitation. The main idea: How about using simulated mean values from meteorological models instead?
A covariate based on the orographic model can be computed at each grid point which has some of the desired properties.

Namely, by calculating the mean of the simulated precipitation in each grid point, which yields a 521 km x 361 km regular grid of covariates.

Observations tend not to be on regular grid points. However, a simple smoother can be used to map the covariates from grid points to observation points.
Triangulation of the domain, Iceland, is constructed

An approximate solutions of the SPDE

$$(\kappa^2 - \Delta)^{\alpha/2} (\tau x(s)) = \mathcal{W}(s), \quad s \in \mathbb{R}^d, \quad \alpha = \nu + d/2, \quad \kappa > 0, \quad \nu > 0$$

can be found on the triangular grid, which is a Gaussian Markov random field (GMRF) representation of a Matérn field

The approximate solution $x(s)$ is represented by

$$x(s) = \sum \psi(s) w_k$$

where $\psi_k$ are piecewise linear in each triangle, and $w_k$ are Gaussian weights.
TRIANGULATION
**Results**

- The following results are based on two MCMC chains, 25,000 iterations each.
- The MCMC chains are inferred with a novel two block updating scheme.
- Runtime is around 4 hours on a laptop (i7 core, 4 Gb ram).
Trace, density and autocorrelation for $\theta$
Two block updating scheme

- Trace, density and autocorrelation for $\theta$
Let $\hat{u}$ denote a posterior estimate for the spatial effects on the *triangulated mesh*.

Since every point in the domain of interest belongs to some triangle, a posterior estimate for the spatial effect can be obtained at every point with a convex linear combination of $\hat{u}$.

Let $\hat{u}_{hd}$ denote a posterior estimate for the spatial effects on the *high resolution grid*. It is obtained by projecting $\hat{u}$ with the linear transformation

$$A\hat{u} = \hat{u}_{hd}$$

where $A$ is a matrix which describes the piecewise linear functions within each triangle.
Location parameter

- Posterior mean of the spatial field $u_\eta$ on the high resolution net.
Location parameter

- Posterior mean of the spatial field $X_\eta \beta_\eta + u_\eta$
Posterior standard deviation of the spatial field $u_\eta$
Posterior mean of the spatial field $u_\nu$
Posterior mean of the spatial field $X_{\nu} + \beta_{\nu} + u_{\nu}$
Posterior standard deviation of the spatial field $u_\nu$
Once the spatial estimates $\eta_{hd}$ and $\tau_{hd}$ at every grid point on the high resolution grid are obtained along with an estimate for $\xi$, an estimate can be given for distribution of maximum annual 24 hour precipitation at every grid point.

In particular, a simple estimate of the $p$-th quantile can be obtained by

$$y_{p,un,j} = \hat{\eta}_{hd,j} + \frac{\exp(\hat{\tau}_{hd,j})}{\hat{\xi}} \left( - \log(p)^{-\hat{\xi}} - 1 \right)$$
ESTIMATION OF THE 95-TH QUANTILE
Conclusions

- The SPDE spatial models yield a computationally efficient prediction scheme.
- The two block sampling scheme is beneficial in terms of:
  - Efficient sampling of the non-likelihood latent parameters regardless of the choice of likelihood.
  - Efficient sampling of the hyper parameters which dramatically reduces autocorrelation.
- Different likelihood functions call for appropriate sampling schemes for the likelihood parameters.


Thank you for your attention